PSE Masters in Economics

Quantitative Macro, Fall 2025

Problem Set #4: Solving the DMP model

Due date: Dec 17 2025, 6pm

By email to: `moritz.scheidenberger@psemail.eu`, with subject `QM1-PS4_NamesOfStudents`.

TA sessions: December 5, 11:00-12:30, R1-13: Intro session. December 12, 11:00-12:30, R1-16: TA support session. December 19, 14:00-15:30, R1-09: Answers session.

Instructions: Please hand in a `.zip` file with the names of your group (e.g. `BROER_SEMPE_PS4.zip`). It should contain a pdf file compiled from LaTeX with your answers and figures, as well as the Matlab and Dynare code that you wrote to solve the problem set, structured as requested below. **All code must be runnable from scratch so that the TA can run and analyse your code, and reproduce your results, if needed.**

You will need to install Dynare (for Matlab) to complete the problem set, and you are allowed to use any built in Matlab functions as needed unless you are explicitly asked not to. Any figure or result shown in the pdf should be interpreted.

You should work in the same group as in the previous problem sets. Please include the names of all group members in the pdf file. You are not allowed to use external codes to solve the problem and should limit your use of AI tools to debugging your own code, when needed.

If you have questions you would like to be answered during the correcting or helping TA sessions, please send them by email to the TA by 4pm the day before the tutorial. If you notice any typo or mistake in the problem set, double check and email immediately Alex Clymo and Moritz Scheidenberger.

Not following the instructions above will lead to penalties in your PS grade.

Good luck!

Purpose of the PS: This problem set focuses on calibrating and solving the DMP model. In part 1 you will first import and treat the data to form your calibration targets. In part 2 you will then calibrate the model in steady state in Matlab, solve the model using Dynare, and produce impulse response functions. In part 4 you will solve the model nonlinearly using an algorithm closely related to value function iteration.

1 Importing data and forming calibration targets

Output: Perform this analysis in a matlab file `main1_data.m` which you should send with your submission. All results should be in Section 1 of your answers pdf file.

I have provided US data on labour market stocks and flows in a Google Sheets spreadsheet available on my website (or here). Download this data as an `xlsx` file.

Question 1: (30% of marks)

1. Steady state targets from monthly data:
   - Import the monthly data from the first tab using Matlab's `readtable` function. Extract the data you need from the table into column vectors, and form an appropriate date vector.
   - Manipulate the data to form the variables needed for your calibration targets:
     Hint: All these operations can be vectorised, for example tightness is `theta = v./u`
     - Compute the unemployment rate as $ur = u/(u + e)$
     - Compute tightness as $\theta = v/u$
     - Compute the job finding rate and separation rates using the Shimer (2005) method. Please read at least Sections I.C and I.D of his paper to learn how to do this properly. The job finding rate is
       $$f_t = 1 - \frac{u_{t+1} - u^s_{t+1}}{u_t} \tag{1}$$
       where u_t is the number of unemployed and u^s_t is the number of unemployed for less than five weeks. The separation rate is
       $$s_t = \frac{u^s_{t+1}}{e_t(1 - 0.5 f_t)} \tag{2}$$
       where e_t is the number of employed workers.
       Hint 1: there is a discontinuity in the u^s_t series that you should correct using the suggestion in Shimer's paper. Hint 2: these operations are not possible in the final period of the sample (since there is no $T + 1$ at the final period T) so leave the final value as NaN.
     - Plot the time series for all four variables and include this in your pdf.
     - Calculate the averages of the unemployment rate, tightness, and the job finding and separation rates over the period 1951M1 to 2019M12 and report these in your pdf. These are our calibration targets.
       Hint: If done correctly, these should correspond closely to the targets reported in the lecture notes.

2. Calibrating the AR(1) process for productivity using quarterly data:

   - Import the quarterly data from the second tab of the Excel file using Matlab's `readtable` function. Extract the labour productivity data into a column vector, and form an appropriate quarterly date vector.

     Hint: There is an option in `readtable` *to choose the tab of the Excel file.*

   - Truncate the data to run from 1951Q1 to 2019Q4. Compute the cyclical component of log productivity using the HP filter with parameter 10^5 using Matlab's `hpfilter` function. Plot log productivity and its trend and cyclical components and include them in your pdf.

   - Compute the standard deviation and autocorrelation of (hp-filtered, log) productivity and report them in your pdf.

3. Create a quarterly series for the unemployment rate and tightness by averaging each three month period. That is, unemployment in 2001Q1 is the average of Jan, Feb, March 2001, 2001Q2 is the average of April, May, June 2001 and so on. Compute the standard deviation and autocorrelation of (quarterly, hp-filtered, log) unemployment and tightness and include in your pdf. Discuss the standard deviations of these series relative to that of productivity in relation to the Shimer puzzle.

   Hint: all these numbers will not be identical to those in Shimer's paper due to the updated sample, but should be in the same ballpark.

4. We will now simulate productivity data from the calibrated AR(1) process assumed in our model to check it matches the properties of productivity in the data. Generate simulated data for the variable y_t from the log AR(1) process

$$\log y_t = (1 - \rho_y) \log y_{ss} + \rho_y \log y_{t-1} + \sigma_y \varepsilon_t$$

   where $y_{ss} = 1$, $\rho_y = 0.975$, $\sigma_y = 0.0051$, ε_t is drawn from the standard normal distribution (mean 0, variance 1) using the `randn` command, one time period is one month, and the simulation runs for $T = 10,000$ months. Store the simulated data in a column vector `ysim`, and fix the seed before the simulation to ensure you get the same results each time. Average the simulated monthly data to make quarterly data (as you did in the last question) and then compute the standard deviation and autocorrelation of (quarterly, hp-filtered, log) productivity from this simulation, and include these in your pdf. If all has gone well, these should be close to the values from the data, confirming that our productivity process matches the data.

2 Calibrating the model and solving the model using Dynare

Output: All results should be in Section 2 of your answers pdf file. Perform this analysis in a matlab file `main2_dynare.m` and associated Dynare file `dmp.mod` which you should send with your submission.

Consider the baseline DMP model with Nash bargaining from the lectures, and focus on the Shimer (2005) calibration. We add functional form assumptions that $M(u,v) = Au^{\alpha}v^{1-\alpha}$ and that y_t follows an AR(1) process. The equations of the model are:

$$U_t = z + \beta \mathrm{E}_t \left[(1 - f_t) U_{t+1} + f_t W_{t+1} \right] \tag{3}$$

$$W_t = w_t + \beta \mathrm{E}_t \left[(1 - s) W_{t+1} + s U_{t+1} \right] \tag{4}$$

$$J_t = y_t - w_t + \beta(1 - s)\mathrm{E}_t \left[J_{t+1} \right] \tag{5}$$

$$W_t - U_t = \frac{\phi}{1 - \phi} J_t \tag{6}$$

$$c = \beta q_t \mathrm{E}_t J_{t+1} \tag{7}$$

$$u_t = \left(1 - f_t\right) u_{t-1} + s(1 - u_{t-1}) \tag{8}$$

$$\theta_t = \frac{v_t}{u_{t-1}} \tag{9}$$

$$f_t = A\theta_t^{1-\alpha} \tag{10}$$

$$q_t = A\theta_t^{-\alpha} \tag{11}$$

$$\log y_t = (1 - \rho_y)\log y_{ss} + \rho_y \log y_{t-1} + \sigma_y \varepsilon_t \tag{12}$$

This gives 10 equations for 10 endogenous variables: $W_t, U_t, J_t, w_t, \theta_t, v_t, u_t, f_t, q_t, y_t$. The productivity innovation $\varepsilon_t \sim N(0, 1)$.

Very important: note the change in timing convention for unemployment here relative to the lectures. Here, the unemployment stock at time t is denoted u_{t-1}, to reflect the fact that it is a state variable. In Dynare, state variables (i.e. variables already fixed at time t) must be dated $t-1$ rather than t. The model behaves identically, but make sure to write up the equations with the correct timing in Dynare.

Non-stochastic steady state: This is the steady state of the model in a world with no productivity shocks ($\varepsilon_t = 0\ \forall t$) so that productivity is constant at y_{ss}. This corresponds to the solution to equations (3) to (12) with expectation operators removed and all variables replaced with steady state values, i.e. $J_t = J_{t+1} = J_{ss}$ and so on.

Question 2: (40% of marks)

Hint: the tasks you are asked to complete below (calibrating the steady state of the model using parameter swapping and calling Dynare) are identical to those performed in example codes for the RBC model that will be made available to you following the first TA session. These codes can be adapted to perform similar steps for the DMP model.

1. Calibrate the model in steady state:
   - Write up the steady state versions of equations (3) to (12) and include them in your pdf.
   - Fix the following parameters: Fix $s = 0.032$, $\beta = 0.95^{\frac{1}{12}}$, $\alpha = 0.5$, and $\phi = \alpha$. Fix the UI replacement rate $\psi = 0.4$, where $z = \psi w_{ss}$. Fix $\rho_y = 0.975$ and $\sigma_y = 0.00507$. Fix the following calibration targets: $u_{ss} = 0.0577$, $\theta_{ss} = 0.655$, and $y_{ss} = 1$.
   - In your pdf document, write up a strategy for calibrating the remaining parameters and solving the remaining steady state variables of the model *analytically* given the fixed parameters and calibration targets. This involves a process called parameter swapping, where rather than solving for all steady state endogenous variables given fixed parameters, we solve for some of the parameters at the same time in order to hit our fixed calibration targets. For example, rather than using the free entry condition to solve for equilibrium steady state tightness, we will use it to reverse engineer the vacancy posting cost needed to achieve the targeted level of tightness by setting c such that $c = \beta q_{ss} J_{ss}$. You should write up a sequence of steps which you can then implement in the code, such that each step can be performed given the steps that came before it. If you are not able to do so, explain how you could instead use a nonlinear equation solver to jointly solve the remaining steady state values and parameters.

     Hint 1: The first step is to use the steady state version of (8) to solve for f_{ss} given s and the targeted u_{ss} as $f_{ss} = s/u_{ss} - s$. Hint 2: the last step is to solve for c using (7) to get $c = \beta q_{ss} J_{ss}$. Hint 3: The hardest part is solving for the wage, value functions, and z. This requires solving a system of simultaneous equations, but you can find an analytical solution for w_{ss} if you do it correctly.
   - Implement this calibration strategy in your Matlab code, and report all steady state values and parameter values in your pdf file. If done correctly, these should correspond closely to those from the lecture notes. Following the Matlab and Dynare file structure from the example RBC codes, save the required parameters and steady state values into `dmp_calibration.mat`.
2. Solve the model using Dynare and produce IRFs:
   - Following the Matlab and Dynare file structure from the example RBC codes, write up your Dynare mod file `dmp.mod` for solving the model using log-linear perturbation. Make sure to be careful to declare all endogenous and exogenous variables and parameters, and to correctly load these parameters from the file `dmp_calibration.mat` that you

created in Matlab. Check that the settings in the `stoch_simul` call at the end of the file are correct for doing a first order loglinear approximation and producing 100-period IRFs.

- If done correctly, you should now be able to calibrate your model, solve the loglinear approximation, and produce IRFs to a positive one standard deviation productivity shock simply by running your file `main2_dynare.m`. Do so, and save the IRF figures and include them in your pdf file. Discuss the response of unemployment to the shock in light of the Shimer puzzle.

3 Solving the DMP model nonlinearly

Output: Perform this analysis in a matlab file `main3_nonlinear.m` which you should send with your submission. All results should be in Section 3 of your answers pdf file.

Consider the Hall (2005) model from the lectures. Letting y be current productivity, we express the tightness block of the model in the state space as

$$U(y) = z + \beta \mathrm{E}_{y'}\left[(1 - f(\theta(y)))U(y') + f(\theta(y))W(y')\right] \tag{13}$$

$$W(y) = \hat{w} + \beta \mathrm{E}_{y'}\left[(1 - s)W(y') + sU(y')\right] \tag{14}$$

$$J(y) = y - \hat{w} + \beta(1 - s)\mathrm{E}_{y'}[J(y')], \tag{15}$$

$$c = \beta\, q(\theta(y))\mathrm{E}_{y'}[J(y')] \tag{16}$$

$$\log y' = (1 - \rho_y) \log y_{ss} + \rho_y \log y + \sigma_y \varepsilon' \tag{17}$$

The value functions $U(y)$, $W(y)$, $J(y)$ and policy function $\theta(y)$ are functions only of current productivity y. Productivity follows a log AR(1) process where the innovation is $\varepsilon' \sim N(0, 1)$. Productivity y is known in the current period, while y' is only revealed next period after the shock ε' is drawn. The wage $\hat{w}$ is a fixed parameter. You should assume the matching function $M(u, v) = \frac{uv}{(u^\iota + v^\iota)^{1/\iota}}$ from den Haan et al. (2000) and Petrosky-Nadeau and Zhang (2017), which implies that $f(\theta) = (1 + \theta^{-\iota})^{-1/\iota}$ and $q(\theta) = (1 + \theta^\iota)^{-1/\iota}$. This is to ensure that $0 \le f(\theta), q(\theta) \le 1$ so that these probabilities remain valid in response to large shocks in our nonlinear solution, which is not guaranteed with the Cobb-Douglas matching function which can produce probabilities in excess of one.

Question 3: (30% of marks)

1. Letting $y_{ss} = 1$, $\rho_y = 0.975$, $\sigma_y = 0.0051$, discretise productivity y using the Rouwenhorst method using the function `rouwen.m` available here with $N = 7$ nodes. This returns a grid of productivity values y_i for $i = 1, \ldots, N$ and associated transition matrix capturing the

probability that $y' = y_j$ given that $y = y_i$. Quickly verify that the productivity grid and transition matrix look sensible and include them in your pdf.

Hint: the `rouwen.m` *function will discretise an AR(1) process in levels, not logs, so you will need to transform the output* `z_Rouw` *to properly account for our log-normal process.*

2. Calibrate the model in steady state under the Shimer (2005) style calibration, as you did in Question 2, to solve for all parameters and steady state values. The only tweak here is that with the new matching function we lose the parameter A. Therefore, do not calibrate $\theta_{ss} = 0.655$ as we did in Question 2, but instead solve for the value of θ_{ss} needed to generate the required value of f_{ss}, assuming that $\iota = 0.5$. Apart from that, the steady state calibration should follow very similar steps, and continue to assume that $\phi = 0.5$. Set the fixed wage equal to the Nash bargained wage in steady state: $\hat{w} = w_{ss}$.

3. Solve equations (13) to (16) nonlinearly to find the functions $U(y)$, $W(y)$, $J(y)$, and $\theta(y)$. These should be values on the productivity grid (e.g. $J_i = J(y_i)$, $\theta_i = \theta(y_i)$) which you store as vectors `J`, `theta`, and so on. Plot these functions, as well as f_i and q_i, and include them in your pdf.

   Hint: You can solve these nonlinear equations however you like (certain parts by value function iteration, for example) but you can make your life much easier by realising that you do not need to solve all four equations simultaneously. Which function can you solve first?

4. Verify whether or not $W_i - U_i \geq 0$ and $J_i \geq 0$ for all i in your solution. What does this imply about the validity of Hall's fixed wage assumption for these parameter values? How does this change as you increase the number of nodes N and why?

References

DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90, 482–498.

HALL, R. E. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95, 50–65.

PETROSKY-NADEAU, N. AND L. ZHANG (2017): "Solving the Diamond–Mortensen–Pissarides Model Accurately," *Quantitative Economics*, 8, 611–650.

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