Heterogeneity in Macro-Labour Models (Part 1)

PSE – Masters Year 2 (M2) – Quantitative Macro 2 (QM2)

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Welcome!

Who am I?

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What are we doing?

- Two lectures studying the role of heterogeneous agents in labour market (search and matching) models, building towards the HANK-SAM model
- One problem set on HANK-SAM, based on material from my and Prof. Broer's lectures

Any questions? Let's go!

Motivation

Heterogeneity in macro labour models

Labour markets are fundamentally heterogeneous.

Workers differ by:

- wages received,
- employment histories,
- assets and savings, ...

Many core macro–labour questions hinge on:

- *Who* experiences which shocks?
- *How* do they respond?
- *How* do individual responses aggregate?

Objective of this lecture: First step to understanding how heterogeneity shapes:

- unemployment and job-finding,
- wage setting and selection,
- welfare and policy evaluation, ...

HANK $\rightarrow$ HANK-SAM

Much of modern heterogeneous-agent macro (HANK) studies:

Incomplete markets + exogenous income risk

- Idiosyncratic income shocks.
- Self-insurance via saving.
- State variables: current income shock and asset position.

This (correctly) captures two core margins of heterogeneity: **income** and **wealth**

By modelling labour markets we can go further:

- What are labour income shocks?
- How do worker and firm choices endogenously shape labour income?

Today:

- Part I: worker's decisions shape their income (McCall, 1970)
- Part II: search and matching + incomplete markets (Krusell et al., 2010)

McCall model (partial equilibrium job acceptance)

Part I: McCall (1970)

Simplest environment generating unemployment from heterogeneity.

Core idea:

- Workers receive random wage offers.
- They optimally choose whether to accept or wait.
- Waiting generates unemployment.

Implications:

- Frictional wage dispersion: identical workers earn different wages.
- Reservation wage summarises optimal behaviour.
- UI shifts the outside option and changes unemployment duration.

Motivation: frictional wage dispersion

Observed fact: Even observably similar workers earn different wages.

Some wage dispersion reflects skill differences, education, occupation, location, ...

But even after controlling for observables, large residual wage dispersion remains.

Abowd et al. (1999): *"Observably equivalent individuals earn markedly different compensation and have markedly different employment histories."*

Hornstein et al. (2011): "Mean min ratio" of residualised wages around 1.5 to 2

Walrasian model $\implies$ Identical workers get paid identical wages...

Search frictions provide an explanation:

- Some randomness in meetings and wage offers
- Past luck and bargaining generate dispersion.

Environment and primitives (McCall, 1970)

Objective: Study optimal job acceptance under wage heterogeneity.

Discrete time $t = 0, 1, 2, \ldots$. Single representative worker (continuum of mass 1), risk neutral, discount factor $\beta \in (0, 1)$. Partial equilibrium, and no aggregate risk.

Labour market states:

- **Unemployed:** receive benefit $b \geq 0$ each period (UI or home production).
- **Employed at wage** w**:** receive wage w each period.

Job offer process:

- If unemployed, receive offer with probability $\lambda \in (0, 1]$.
- Wage draw $w \sim F$, continuous on $[0, \infty)$.

Job destruction:

- If employed, separation occurs with probability $\delta \in (0, 1)$.

Key ingredients:

- Wage heterogeneity (via F)
- Idiosyncratic employment risk via random job offer arrivals and separations

Timing and decisions

If unemployed at start of period:

1. Receive benefit b.
2. With probability λ, draw $w \sim F$.
3. If offer arrives, choose accept or reject.

If employed at wage w:

1. Receive wage w.
2. With probability δ, job ends and worker becomes unemployed next period.

Observations:

- Decision problem (accept / reject) only for unemployed when receive an offer.
- Unemployment due to infrequent arrival of “good enough” offers

Value functions: worker problem

Define:

- U: expected discounted value of being unemployed.
- $W(w)$: expected discounted value of being employed at wage w.

These satisfy:

$$W(w) = w + \beta \Big[(1-\delta) W(w) + \delta U \Big]$$

$$U = b + \beta \Big[(1-\lambda) U + \lambda \, \mathrm{E} \left[\max\{ W(w), U \} \right] \Big]$$

Interpretation:

- $W(w)$: current wage + discounted continuation value, accounting for job destruction.
- U: current benefit + option value of future offers. Expectation is over offers from $F(w)$

Key structure:

- $W(w)$ is increasing in $w \implies$ optimal policy is characterised by a reservation wage.
- Note: proper notation should allow workers to quit if $W(w) < U$, ignore for clarity

Solving for $W(w)$

Start from:

$$W(w) = w + \beta\Big[(1-\delta)W(w) + \delta U\Big]$$

Rearrange:

$$\boxed{W(w) = \frac{w + \beta\delta U}{1-\beta(1-\delta)}} \qquad \text{where } 1-\beta(1-\delta) = 1-\beta+\beta\delta > 0$$

Implications:

- $W(w)$ is linear and strictly increasing in w.
- Higher separation risk ($\delta \uparrow$) reduces the slope.
- Employed value depends on outside option U.

Reservation wage and acceptance decision

Definition: The reservation wage w_r makes the worker indifferent between employment and unemployment:

$$\boxed{W(w_r) = U}$$

Decision rule:

$$\boxed{\text{Accept iff } w \geq w_r}$$

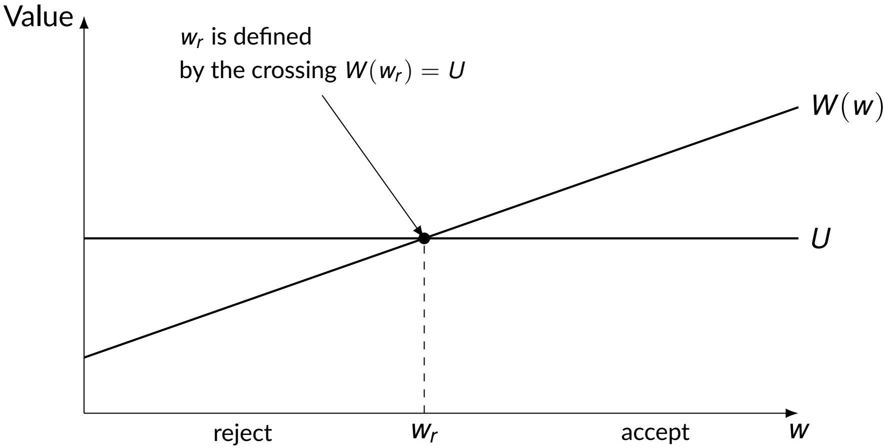
Why a cutoff rule?

- $W(w)$ is strictly increasing in w.
- U does not depend on w.
- Therefore, indifference occurs at a unique crossing.

Continuity of F:

- Because F is continuous, the cutoff is exact.

McCall: reservation wage graphically



Accept iff $W(w) \geq U \iff w \geq w_r$.

A useful result: linking U and w_r

Start from the reservation wage definition:

$$W(w_r) = U.$$

Using the closed form

$$W(w) = \frac{w + \beta\delta U}{1 - \beta(1-\delta)},$$

evaluate at w_r and solve for U.

$$\boxed{U = \frac{w_r}{1-\beta}.}$$

Interpretation:

- The value of unemployment equals the capitalised value of the reservation wage.
- Higher patience ($\beta \uparrow$) increases the value of waiting.
- Why this helps? Eliminate $U \rightarrow$ model reduces to a single fixed point equation in w_r.

Appendix: derive $U = \frac{w_r}{1-\beta}$

Reservation wage fixed point

Impose the cutoff rule inside the unemployed Bellman equation. Use:

- Accept iff $w \geq w_r$
- $U = \dfrac{w_r}{1-\beta}$

After simplification:

$$\boxed{w_r = b + \frac{\beta\lambda}{1-\beta(1-\delta)} \int_{w_r}^{\infty} (w - w_r)\, \mathrm{d}F(w)}$$

Economic interpretation:

- b = value of accepting zero wage (outside option).
- The integral term = expected gain from waiting for better offers.
- λ scales how often this option arrives.
- $1-\beta(1-\delta)$ captures discounting and expected length of match.

Key idea: Reservation wage = current benefit + option value of search.

Appendix: full derivation

Reservation wage fixed point

Using integration by parts:

$$\int_{w_r}^{\infty} (w - w_r)\, \mathrm{d}F(w) = \int_{w_r}^{\infty} \left(1 - F(w)\right)\, \mathrm{d}w.$$

So equivalently:

$$\boxed{w_r = b + \frac{\beta\lambda}{1 - \beta(1 - \delta)} \int_{w_r}^{\infty} \left(1 - F(w)\right)\, \mathrm{d}w.}$$

Why this representation is useful:

- The dependence on w_r is entirely in the lower limit.
- Derivative is immediate:

$$\frac{d}{dw_r} \int_{w_r}^{\infty} (1 - F(w))\, dw = -(1 - F(w_r)).$$

This makes comparative statics easier.

Comparative statics via implicit differentiation

Define $A(\beta, \lambda, \delta) = \frac{\beta\lambda}{1-\beta(1-\delta)}$ and $T(w_r) = \int_{w_r}^{\infty} (1 - F(w))\, dw$

Then the fixed point becomes: $w_r = b + A\, T(w_r)$

To find dw_r / db, let $w_r = w_r(b)$ and differentiate whole equation wrt b: Appendix: derivation

$$\frac{dw_r}{db} = \frac{1}{1 - AT'(w_r)} = \frac{1}{1 + A(1 - F(w_r))} > 0.$$

So raising UI benefits raises the reservation wage ($b \uparrow \Rightarrow w_r \uparrow$)

Economic interpretation:

- Higher UI shifts the outside option upward.
- Workers become more selective.
- Reservation wage increases.

Similar logic implies:

- Faster arrival rate raises the reservation wage ($\lambda \uparrow \Rightarrow w_r \uparrow$)
- More patience raises the reservation wage ($\beta \uparrow \Rightarrow w_r \uparrow$)

Partial-equilibrium unemployment

Even in this simple partial-equilibrium environment, the model implies a well-defined steady-state unemployment rate.

Two transition rates:

- **EU (job destruction):** An employed worker separates with probability δ.
- **UE (job finding):** An unemployed worker receives an acceptable offer with probability

$$\lambda \times \mathbb{P}(w \geq w_r) = \lambda\big(1 - F(w_r)\big).$$

Steady-state flow balance: Outflows from employment must equal inflows:

$$(1-u)\delta = u\,\lambda(1 - F(w_r)).$$

Solving,

$$\boxed{u = \frac{\delta}{\delta + \lambda(1 - F(w_r))}.}$$

Key idea: Worker selectivity matters: higher w_r lowers the UE rate.

How UI affects unemployment in McCall (partial equilibrium)

Step 1: Effect on reservation wage. From the w_r fixed point, $b \uparrow \Rightarrow w_r \uparrow$.

Step 2: Effect on job finding. Higher w_r reduces the acceptance probability, $1 - F(w_r) \downarrow$, therefore the UE rate falls:

$$\lambda(1 - F(w_r)) \downarrow .$$

Step 3: Effect on steady-state unemployment. Since unemployment equals separation divided by total turnover,

$$u = \frac{\delta}{\delta + \lambda(1 - F(w_r))},$$

a lower UE rate implies $u \uparrow$.

Mechanism summary: $b \uparrow \Rightarrow w_r \uparrow \Rightarrow$ acceptance $\downarrow \Rightarrow$ UE rate $\downarrow \Rightarrow u \uparrow$.

Next section: general equilibrium. In McCall, λ is exogenous. In matching models, policy can also affect λ through vacancy creation and market tightness.

KMS (2010): from PE to GE + incomplete markets

Krusell–Mukoyama–Şahin (2010)

What McCall abstracts from:

- Firms and vacancy creation.
- Endogenous job-finding rates.
- Asset accumulation and precautionary savings.
- Business cycle shocks.

Krusell–Mukoyama–Şahin (2010):
First paper to combine:

- Search and matching (DMP),
- Incomplete markets (Bewley/Aiyagari),
- Aggregate shocks (Krusell–Smith).
- Note: but abstracts from wage offer distribution: only one type of job

Key question 1: How does adding incomplete markets affect the DMP model?

Key question 2: How does unemployment insurance affect equilibrium unemployment and welfare when workers can self-insure and firms create jobs?

Environment I (steady state, no aggregate risk)

To understand the mechanism clearly, first shut down aggregate risk.

Workers *[continuum of measure 1]*

- Each worker is either employed or unemployed.
- Can self-insure by saving in capital or firm equity

Firms *[large measure of potential firms]*

- Each firm employs at most one worker. Unmatched firms post vacancies to hire.
- Firms rent capital competitively at rental rate r

Assets and financial structure

- Workers cannot trade shares in individual firms, they hold an aggregate stock market bundle (or equivalently a representative firm) with price p and dividends d per share
- Both capital and equity are riskless assets $\Rightarrow$ by arbitrage:

$$\text{capital return} = \text{equity return} \equiv \frac{1}{q}$$

- Therefore can use total wealth $a \equiv (1 + r - \delta)k + (p + d)x$ as a state variable

Environment II (steady state, no aggregate risk)

Production: Each match produces output

$$zF(k), \quad F' > 0,\ F'' < 0.$$

Firms rent capital at competitive rental rate r *(note: r is not the interest rate or total return)*

$$r = zF'(k).$$

Labour market matching: *[assumes exogenous separations]*

Tightness: $\theta = \frac{v}{u}$, worker job finding rate: $\lambda_w(\theta)$, firm vacancy filling rate: $\lambda_f(\theta)$

Law of motion for unemployment:

$$u' = (1 - \lambda_w(\theta))u + \sigma(1 - u).$$

Good market: Output used for consumption, investment, vacancy posting costs

Households: incomplete markets + search risk

Workers face idiosyncratic unemployment risk and cannot insure it fully. $a' \geq \underline{a}$

Unemployed value, $U(a)$:

$$U(a) = \max_{a' \geq \underline{a}} \left\{ u(c) + \beta[(1-\lambda_w)U(a') + \lambda_w W(a')] \right\}$$

where

$$c + qa' = a + h$$

and $W(a)$ is the value of being employed at the equilibrium bargained wage.

Employed value today before wage bargain is finalised, $\widetilde{W}(w, a)$:

$$\widetilde{W}(w, a) = \max_{a' \geq \underline{a}} \left\{ u(c) + \beta[(1-\sigma)W(a') + \sigma U(a')] \right\}$$

where

$$c + qa' = a + w$$

and it is understood that next period you will receive tomorrow's bargained wage

Wage determination: wealth-dependent bargaining

Wages solve Nash bargaining, where V is vacancy value and $\widetilde{J}(w,a)$ is firm value pre-bargaining:

$$\max_{w} \left(\widetilde{W}(w,a) - U(a)\right)^{\gamma} \left(\widetilde{J}(w,a) - V\right)^{1-\gamma}.$$

Result: $w = \omega(a)$

Equilibrium employed value: $W(a) = \widetilde{W}(\omega(a), a)$

Why does bargained wage depend on assets?

- Wealthier workers have higher outside option because unemployment is less painful
- Therefore bargain for slightly higher wages.

BUT, quantitative finding: Wage dispersion from asset heterogeneity is small. Picture

Free entry and tightness

Firm value, $\widetilde{J}(w, a)$, depends on the assets a of the worker they employ. Why?

As with worker value, let $J(a) = \widetilde{J}(\omega(a), a)$ be equilibrium firm value

Free entry condition: $V = 0$

Vacancy posting cost ξ. Firms post vacancies until expected value, V, is zero. Implies:

$$\xi = q\lambda_f(\theta) \int J(\psi_u(a)) \frac{f_u(a)}{u} da.$$

- q: discounting since workers start work next period
- Integral over distribution of unemployed workers, $f_u(a)$, since could meet anyone
- $\psi_u(a)$ is unemployed policy function $a' = \psi_u(a)$, to compute $J(a')$

Key GE mechanism: Higher wages $\Rightarrow$ lower surplus $J(a)$ $\Rightarrow$ lower vacancy posting.

Steady state equilibrium

Equilibrium requires solving for prices (q, θ) and a wage function $\omega(a)$ such that:

1. q clears the asset market:

- Total asset demand: $A^d = \int af_u(a)da + \int af_e(a)da$
- Capital supply: $1/q = 1 + r - \delta \implies r = 1/q - 1 + \delta \implies$ capital supply K^s
- Equity supply: $1/q = \frac{d+p}{p} \implies p = d/(1/q - 1)$ where d is aggregate dividend payment. Total supply of shares normalised to $X = 1$
- Integrating asset definition $a \equiv (1 + r - \delta)k + (p + d)x$ across all workers gives our market clearing condition

$$A^d = (1 + r - \delta)K^s + p + d$$

where we used $\int kf_u(a)da + \int kf_e(a)da = K^s$ and $\int xf_u(a)da + \int xf_e(a)da = X = 1$

2. θ from free entry condition

3. $\omega(a)$ from Nash bargaining condition

How UI affects equilibrium aggregates in KMS

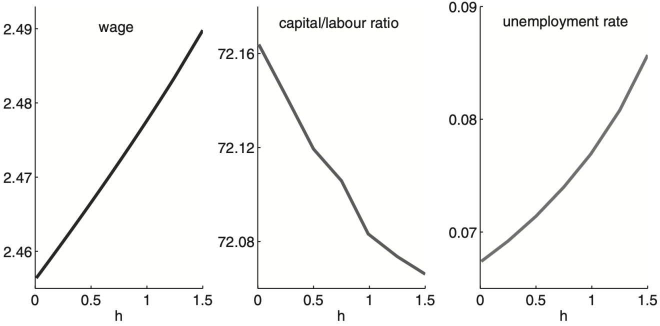


FIGURE 2

Wages and capital/labour ratio and the unemployment rate as a function of *h*

Raising UI benefits, *h*:

- Raises the wage via Nash bargaining
- Lowers the K/L ratio by reducing the need for self insurance so reducing saving in capital
- Raises the unemployment rate as higher wage lowers profits and hence vacancy posting

[Note: Calibration follows Shimer (2005), and they also explore a Hagedorn and Manovskii (2008) calibration]

Main quantitative result: optimal UI

Optimal UI balances gains from insurance against distortions to production. KMS compute steady-state welfare (consumption equivalents, λ) for different groups, across h values:

TABLE 2

Average values of λ, compared with the benchmark of $h = 0.99$

h from 0.99 to	Total (%)	Unemployed (%)	Employed (%)	% gaining	Poorest unemployed (%)	Poorest employed (%)
0.01	0.11	−0.09	0.13	92.10	−4.0	−1.0
0.25	0.12	−0.02	0.14	92.29	−1.2	−0.3
0.50	0.11	0.01	0.12	99.22	−0.54	−0.09
0.75	0.07	0.02	0.07	99.96	−0.2	0.00
1	0	0	0	0	0	0
1.25	−0.14	−0.09	−0.15	0.00	0.06	−0.1
1.50	−0.38	−0.27	−0.39	0.00	−0.04	−0.32

Results:

- Average welfare maximised around $h = 0.25$, implying a $\simeq 10\%$ replacement rate
- Why? Insurance gains exist but are modest, compared to large vacancy distortion *(realistic?)*
- Unemployed suffer from low h, but employed gain (and unemployed will eventually employed again)

Contrast with Aiyagari: With exogenous worker transition rates, full insurance ($h = w$) is optimal because there is no vacancy distortion. $\Longrightarrow$ modelling the labour market is important for HA models!

Adding aggregate productivity shocks

KMS finally introduce aggregate productivity: $z \in \{g, b\}$ with a Markov transition matrix. Compare model with incomplete markets ("incomplete") to standard representative agent DMP model ("linear"):

TABLE 5

Summary statistics for the linear model and for the incomplete-markets model. The entries for u are the levels, and the other entries are percentage changes from z = 1.00

		y	u (%)	v	θ	$\bar{k}$
Shimer calibration	$z = 0.98$-linear	−2.94	7.79	−3.6	−4.8	−3.0
	$z = 0.98$-incomplete	−2.94	7.79	−3.6	−4.8	−3.0
	$z = 1.02$-linear	+2.97	7.60	+3.6	+4.9	+3.0
	$z = 1.02$-incomplete	+2.97	7.60	+3.6	+4.9	+3.0
HM calibration	$z = 0.98$-linear	−11.2	16.10	−21.4	−62.0	−11.6
	$z = 0.98$-incomplete	−11.2	16.10	−21.3	−61.8	−11.6
	$z = 1.02$-linear	+4.0	6.23	+21.5	+52.4	+4.6
	$z = 1.02$-incomplete	+4.0	6.23	+21.6	+52.4	+4.6

Results:

- Adding incomplete markets barely changes the DMP model! In particular, unemployment volatility does not change at all $\implies$ incomplete markets alone are not a solution to the Shimer (2005) puzzle
- Why? Nash bargaining $\implies$ real wages are still flexible. More deeply, *why* should we expect incomplete markets to amplify recessions in this model? What if we added some sticky prices? (Next week!)

Summary and next week

Summary and next week

Today: *(HA+SAM)*

- Macro labour models naturally force us to think about heterogeneity
- McCall: wage heterogeneity $\Rightarrow$ theory of reservation wages
- KMS: add incomplete markets to DMP to study insurance vs job creation

Next week: *(HANK+SAM)*

- HANK: incomplete markets + nominal rigidities. HANK-SAM adds labour-market block
- KMS says heterogeneity doesn't amplify recessions, but HANK says it does. HANK-SAM says what?

Reading List (Part 1)

Core Textbooks and required reading

Core textbooks: *[not required reading, but useful resources]*

- Pissarides (2000): *Equilibrium Unemployment Theory (2nd ed)*. Classic DMP reference.
- Ljungqvist and Sargent (2018): *Recursive Macroeconomic Theory (4th ed)*.
- Cahuc et al. (2014): *Labor Economics (2nd ed)*. Comprehensive textbook on labor economics.
- Quant Econ coding course website, including McCall model in Julia and Python.

The following papers are considered required reading for lectures 1 and 2:

- Krusell et al. (2010) *Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations*
- Ravn and Sterk (2021) *Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach*
- *[no need to memorise or perfectly understand everything, but you must at least read these]*

References I

- Abowd, John M., Francis Kramarz, and David N. Margolis (1999). "High Wage Workers and High Wage Firms". In: *Econometrica* 67.2, pp. 251–333. ISSN: 0012-9682. JSTOR: 2999586. (Visited on 02/16/2026).
- Cahuc, Pierre, Stephane Carcillo, and Andre Zylberberg (2014). *Labor Economics, Second Edition*. Cambridge, MA: The MIT Press. ISBN: 978-0-262-02770-0.
- Hagedorn, Marcus and Iourii Manovskii (Sept. 2008). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited". In: *American Economic Review* 98.4, pp. 1692–1706. ISSN: 0002-8282. DOI: 10.1257/aer.98.4.1692. (Visited on 04/10/2025).
- Hornstein, Andreas, Per Krusell, and Giovanni L. Violante (Dec. 2011). "Frictional Wage Dispersion in Search Models: A Quantitative Assessment". In: *American Economic Review* 101.7, pp. 2873–2898. ISSN: 0002-8282. DOI: 10.1257/aer.101.7.2873. (Visited on 08/02/2024).

References II

- Krusell, Per, Toshihiko Mukoyama, and Ayşegül Şahin (Oct. 2010). "Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations". In: *The Review of Economic Studies* 77.4, pp. 1477–1507. ISSN: 0034-6527. DOI: 10.1111/j.1467-937X.2010.00700.x. (Visited on 07/10/2022).
- Ljungqvist, Lars and Thomas J. Sargent (Sept. 2018). *Recursive Macroeconomic Theory*. 4th ed. Cambridge, MA, USA: MIT Press. ISBN: 978-0-262-03866-9.
- McCall, J. J. (1970). "Economics of Information and Job Search". In: *The Quarterly Journal of Economics* 84.1, pp. 113–126. ISSN: 0033-5533. DOI: 10.2307/1879403. JSTOR: 1879403. (Visited on 07/15/2024).
- Pissarides, Christopher A. (Mar. 2000). *Equilibrium Unemployment Theory*. 2nd ed. Cambridge, MA, USA: MIT Press. ISBN: 978-0-262-53398-0.
- Ravn, Morten O and Vincent Sterk (Apr. 2021). "Macroeconomic Fluctuations with HANK & SAM: An Analytical Approach". In: *Journal of the European Economic Association* 19.2, pp. 1162–1202. ISSN: 1542-4766. DOI: 10.1093/jeea/jvaa028. (Visited on 07/10/2022).

References III

Shimer, Robert (Feb. 2005). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies". In: *American Economic Review* 95.1, pp. 25–49. ISSN: 0002-8282. DOI: 10.1257/0002828053828572. (Visited on 07/04/2022).

Appendix: McCall model

Derivation: $U = \frac{w_r}{1-\beta}$

We have

$$W(w) = \frac{w + \beta\delta U}{1 - \beta(1-\delta)}.$$

Indifference at the reservation wage: $W(w_r) = U$:

$$U = \frac{w_r + \beta\delta U}{1 - \beta(1-\delta)}.$$

Multiply by $1 - \beta(1-\delta) = 1 - \beta + \beta\delta$:

$$U(1 - \beta + \beta\delta) = w_r + \beta\delta U.$$

Cancel $\beta\delta U$ from both sides:

$$U(1-\beta) = w_r \quad \Rightarrow \quad \boxed{U = \frac{w_r}{1-\beta}.}$$

Return

Derivation: reservation wage fixed point

Unemployed Bellman: $U = b + \beta\Big[(1-\lambda)U + \lambda\ \mathrm{E}[\max\{W(w), U\}]\Big]$

With a cutoff w_r: $\mathrm{E}[\max\{W(w), U\}] = \int_0^{w_r} U\ \mathrm{d}F(w) + \int_{w_r}^{\infty} W(w)\ \mathrm{d}F(w) = UF(w_r) + \int_{w_r}^{\infty} W(w)\ \mathrm{d}F(w)$

Substitute: $U = b + \beta\Big[U(1-\lambda+\lambda F(w_r)) + \lambda\int_{w_r}^{\infty} W(w)\ \mathrm{d}F(w)\Big]$

Rearrange: $U\Big(1-\beta(1-\lambda+\lambda F(w_r))\Big) = b + \beta\lambda\int_{w_r}^{\infty} W(w)\ \mathrm{d}F(w)$

Use $W(w) - U = \dfrac{w - w_r}{1-\beta(1-\delta)}$ for $w \geq w_r$ (since $W(w_r) = U$), hence:

$$\int_{w_r}^{\infty} W(w)\ \mathrm{d}F(w) = U(1-F(w_r)) + \frac{1}{1-\beta(1-\delta)}\int_{w_r}^{\infty}(w - w_r)\ \mathrm{d}F(w).$$

Plugging in and simplifying cancels the common U terms and yields:

$$U(1-\beta) = b + \frac{\beta\lambda}{1-\beta(1-\delta)}\int_{w_r}^{\infty}(w - w_r)\ \mathrm{d}F(w).$$

Finally use $U(1-\beta) = w_r$:

$$\boxed{w_r = b + \frac{\beta\lambda}{1-\beta(1-\delta)}\int_{w_r}^{\infty}(w - w_r)\ \mathrm{d}F(w).}$$

Return

Integration by parts

Let $S(w) = 1 - F(w)$ be the survival function, so $\mathrm{d}S(w) = -\mathrm{d}F(w)$. Define

$$I(w_r) = \int_{w_r}^{\infty} (w - w_r)\ \mathrm{d}F(w) = -\int_{w_r}^{\infty} (w - w_r)\ \mathrm{d}S(w).$$

Use integration by parts (Stieltjes) with $u(w) = w - w_r$, $\mathrm{d}v = \mathrm{d}S(w)$ so $\mathrm{d}u = \mathrm{d}w$, $v = S(w)$:

$$\int_{w_r}^{\infty} (w - w_r)\ \mathrm{d}S(w) = (w - w_r)S(w)\Big|_{w_r}^{\infty} - \int_{w_r}^{\infty} S(w)\ \mathrm{d}w.$$

Boundary term is 0 (at w_r it is 0; as $w \to \infty$, $S(w) \to 0$ and the product vanishes under mild conditions). Thus:

$$\int_{w_r}^{\infty} (w - w_r)\ \mathrm{d}S(w) = -\int_{w_r}^{\infty} (1 - F(w))\ \mathrm{d}w.$$

Multiply by -1:

$$\boxed{\int_{w_r}^{\infty} (w - w_r)\ \mathrm{d}F(w) = \int_{w_r}^{\infty} (1 - F(w))\ \mathrm{d}w.}$$

Return

Appendix: Deriving $\frac{dw_r}{db}$

Can also define equation as $G(w_r; b) = 0$, where $G(w_r; b) = w_r - b - AT(w_r)$
Differentiate total derivative:

$$\frac{dG}{db} = \frac{\partial G}{\partial w_r}\frac{dw_r}{db} + \frac{\partial G}{\partial b} = 0.$$

Compute partial derivatives:

$$\frac{\partial G}{\partial b} = -1 \qquad \frac{\partial G}{\partial w_r} = 1 - AT'(w_r).$$

Since $T'(w_r) = -(1 - F(w_r))$, we obtain:

$$\frac{\partial G}{\partial w_r} = 1 + A(1 - F(w_r)).$$

Thus:

$$\boxed{\frac{dw_r}{db} = \frac{1}{1 + A(1 - F(w_r))} > 0.}$$

Return

Appendix: KMS Model

KMS2010 wages as a function of assets ▸ Return

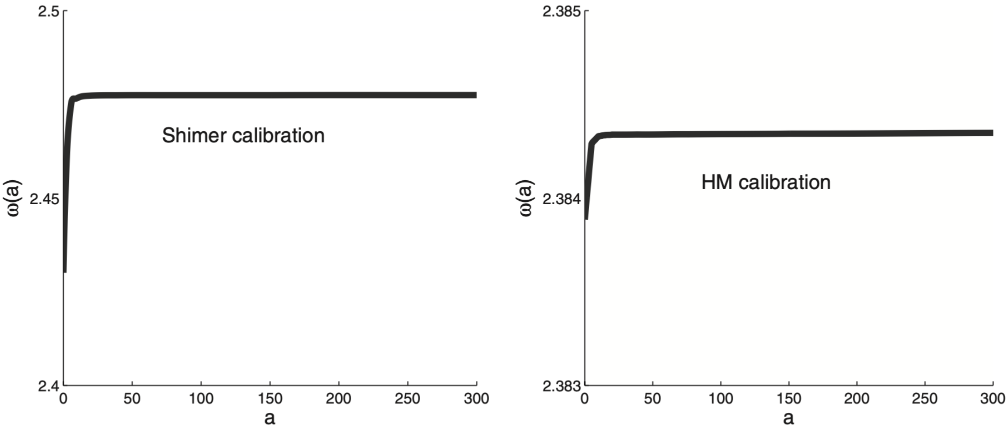


FIGURE 1

Wage function for the Shimer (left panel) and the HM (right panel) calibrations