# Heterogeneity in Macro-Labour Models (Part 2)

## PSE – Masters Year 2 (M2) – Quantitative Macro 2 (QM2)

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# From HA+SAM to HANK+SAM

# Where we left off (Lecture 1)

- McCall (1970): worker job acceptance model (no incomplete markets)
- KMS (2010): incomplete markets + search (DMP meets Krusell Smith)
- Workers face idiosyncratic unemployment risk.
- Asset heterogeneity matters for *welfare*.
- But KMS result:

  Incomplete markets have small effects on unemployment volatility.

- Surplus logic still dominates fluctuations.

**Puzzle:** If heterogeneity does not amplify volatility in KMS, why is it central in HANK?

# The apparent contradiction

- KMS (2010):
  - Old school calibration, not targeting high MPCs
  - No New Keynesian amplification via sticky prices and demand effects
- HANK literature:
  - Calibrations / features to generate high MPCs (e.g. two asset model / high discount rates)
  - Amplification via New Keynesian demand effects
  - Heterogeneity central for aggregate dynamics.

**Today:** Combine HANK + SAM.

**Why is this useful?** Search frictions endogenise (countercyclical) income risk in HANK.

# Core mechanisms of HANK+SAM

Consider a monetary tightening as an example:

$$\downarrow \text{Demand} \;\Rightarrow \downarrow \text{Vacancies} \;\Rightarrow \downarrow \text{Job finding } (\eta) \;\Rightarrow$$

**Mechanism 1: MPCs $<<<$ 1:**

$$\Rightarrow \uparrow \text{Unemployment} \;\Rightarrow \downarrow \text{Income} \;\Rightarrow \downarrow \text{Spending (since } \mathit{MPC} < 1) \;\Rightarrow \downarrow \text{Demand}$$

**Mechanism 2: Endogenous countercyclical income risk:**

$$\Rightarrow \uparrow \text{Unemployment risk} \;\Rightarrow \uparrow \text{Precautionary saving} \;\Rightarrow \downarrow \text{Demand}$$

**Note:** both of these are present in HANK, but endogenised in HANK-SAM

# Ravn & Sterk (2021): Analytical HANK+SAM

# Why Ravn & Sterk?

- Analytical HANK + SAM model.
- Clever trick means no asset distribution in equilibrium (zero-liquidity limit)...
- ... but precautionary behaviour survives and affects the equilibrium
- Nice graphical intuitions in steady-state

**This lecture:**

1. Use Ravn & Sterk to build intuition
2. Briefly discuss fully-fledged models with asset distribution at end of lecture
3. In problem set, you will solve a full HANK+SAM with asset distribution

# Model structure

- **Households:**
  - Workers face unemployment risk, capitalists own firms
  - Zero liquidity in equilibrium $\implies$ consumption equals income.
- **Labour market (SAM):**
  - Matching function.
  - Job finding rate $\eta$.
  - Wages from Nash bargaining.
- **Firms:**
  - Produce using only labour, and hiring costs enter marginal cost.
  - Monopolistic competition + sticky prices via Rotemberg friction $\implies$ NKPC
- **Nominal block:**
  - Sticky prices via Rotemberg pricing.
  - Central bank with a Taylor rule.

**Important note:** Their paper starts with a very general notation and model, which is later specialised. I simplify the notation and structure (imposing strict worker / capitalist separation from Assumption 1) from the start to ease exposition, but the model is the same.

# Environment

Time discrete, $t = 0, 1, 2, \dots$

Aggregate state: $X_t$, contains aggregate productivity shock $A_t$ and worker distribution

Two household types:

- **Workers** (measure $1 - \xi$):
  - Can work.
  - Face unemployment risk.
  - Trade bonds.
  - Cannot hold equity.
- **Capitalists** (measure $\xi$):
  - Cannot work.
  - Own firms.
  - Trade bonds.

Goal: understand how labour market risk feeds into aggregate demand.

# Labour Market (Search and Matching)

Matching function:

$$m_t = e_t^{\alpha} v_t^{1-\alpha}$$

with tightness $\theta_t = \frac{v_t}{e_t}$ and job finding rate:

$$\eta_t = \frac{m_t}{e_t} = \theta_t^{1-\alpha}$$

Vacancy filling rate:

$$q_t = \frac{m_t}{v_t} = \theta_t^{-\alpha}$$

Employment evolves according to:

$$n_t = (1-\omega)n_{t-1} + \eta_t e_t$$

where $u_t = 1 - \xi - n_{t-1}$ is number unemployed, and where $e_t$ is the number of searchers this period:

$$e_t = u_t + \omega n_{t-1}$$

Notice the assumption: **you can search for a job the same period you lose a job.**

# Workers: Preferences and Budget

Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_t^{1-\mu}}{1-\mu} - \zeta \cdot \mathbf{1}\{\text{employed}\} \right)$$

Income:

$$y_t^{lab} = \begin{cases} w_t & \text{if employed} \\ \vartheta & \text{if unemployed} \end{cases}$$

Budget constraint (real):

$$c_t + \frac{b_t}{R_t} = y_t^{lab} + \frac{b_{t-1}}{\Pi_t}$$

Borrowing constraint:

$$b_t \geq -\psi w_t \text{ if employed}, \quad b_t \geq 0 \text{ if unemployed}$$

Workers face *uninsured idiosyncratic employment risk*.

Take the real wage $w_t$ as given for now, but it is set via Nash bargaining in the paper (we will later just assume fixed for simplicity)

# Worker Problem (Recursive Form)

Let $V^n(b, X)$ and $V^u(b, X)$ denote value functions given current savings $b$ and aggregate state $X$

**Unemployed:**

$$V^u(b, X) = \max_{c,b'} \left\{ \frac{c^{1-\mu}}{1-\mu} + \beta E\Big[\eta V^n(b', X') + (1-\eta) V^u(b', X')\Big] \right\}$$

$$c + \frac{b'}{R(X)} = \vartheta + \frac{b}{\Pi(X)}, \qquad b' \geq 0$$

**Employed:**

$$V^n(b, X) = \max_{c,b'} \left\{ \frac{c^{1-\mu}}{1-\mu} - \zeta + \beta E\Big[(1-\omega(1-\eta'))V^n(b', X') + \omega(1-\eta') V^u(b', X')\Big] \right\}$$

$$c + \frac{b'}{R(X)} = w(X) + \frac{b}{\Pi(X)}, \qquad b' \geq -\psi w(X).$$

where $\eta = \eta(X)$. Note that $\omega(1-\eta')$ is the probability of being unemployed tomorrow. *Why?*

# Worker Euler Conditions

First-order conditions with Kuhn–Tucker multiplier inequalities.

**Unemployed:**

$$c_{u,t}^{-\mu} \geq \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} \left( (1-\eta_t) c_{u,t+1}^{-\mu} + \eta_t c_{n,t+1}^{-\mu} \right) \right]$$

**Employed:**

$$c_{n,t}^{-\mu} \geq \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} \left( \omega(1-\eta_{t+1}) c_{u,t+1}^{-\mu} + (1-\omega(1-\eta_{t+1})) c_{n,t+1}^{-\mu} \right) \right]$$

In general we would have our usual policy functions

$$c^n = c^n(b, X), \qquad c^u = c^u(b, X),$$

and incomplete markets would $\Rightarrow$ a non-degenerate asset distribution. **BUT** a trick will later ensure $b = 0$ for all workers.

# Capitalists

Cannot work, but own all firms. Preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{c,t}^{1-\mu}}{1-\mu}$$

Budget:

$$c_{c,t} + \frac{b_t^c}{R_t} = d_t + \frac{b_{t-1}^c}{\Pi_t} + \vartheta$$

Borrowing constraint forbids borrowing in bonds (important): $b_t^c \geq 0$

Euler equation for bonds:

$$c_{c,t}^{-\mu} \;\geq\; \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} c_{c,t+1}^{-\mu} \right]$$

Stochastic discount factor for pricing decisions of any firms they own:

$$\Lambda_{t,t+1} = \beta \left( \frac{c_{c,t+1}}{c_{c,t}} \right)^{-\mu} .$$

# Zero Liquidity Equilibrium (the clever trick of this paper)

**Key assumptions:**

1. Worker households can only save in bonds, not equity
2. Unemployed (and capitalists) cannot borrow due to $b' \geq 0$ constraint

But to save in bonds, someone else must be borrowing. Bond market clearing:

$$B_t^u + B_t^n + B_t^c = 0.$$

where I let $B_t^x$ be total bond holdings of unemployed, employed, and capitalists

We immediately see that $B_t^u, B_t^c \geq 0$ and $B_t^u + B_t^n + B_t^c = 0$ logically implies either

1. Employed weakly want to save ($B_t^n \geq 0$), requiring $B_t^u = B_t^n = B_t^c = 0$, or
2. Employed want to borrow ($B_t^n < 0$), requiring $B_t^u, B_t^c \geq 0$, one with strict inequality

**Ravn & Sterk prove that $B_t^u = B_t^n = B_t^c = 0$ is the unique equilibrium. Key idea:**

- Since you save by lending, if non-employed can't borrow, then employed can't save!
- How can this be an equilibrium? Prices need to adjust so that $b' = 0$ is optimal for every agent.
- Every agent is either i) strictly borrowing constrained, or ii) optimally chooses $b' = 0$

# Consumption Equals Income ‣ proof intuition

Zero liquidity $\Rightarrow$ no borrowing or saving and all agents consume their income:

$$c_{n,t} = w_t, \qquad c_{u,t} = \vartheta, \qquad c_{c,t} = \frac{1}{\xi}\left(y_t - \kappa v_t - w_t n_t - \frac{\varphi}{2}(\Pi_t - 1)^2 y_t\right) + \vartheta.$$

All agents are fully hand to mouth, but as an equilibrium outcome.

Must be consistent with all three bond Eulers with consumption replaced with income:

$$c_{u,t}^{-\mu} \ \geq \ \beta E_t\left[\frac{R_t}{\Pi_{t+1}}\left((1-\eta_t)c_{u,t+1}^{-\mu} + \eta_t c_{n,t+1}^{-\mu}\right)\right]$$

$$c_{n,t}^{-\mu} \ \geq \ \beta E_t\left[\frac{R_t}{\Pi_{t+1}}\left(\omega(1-\eta_{t+1})c_{u,t+1}^{-\mu} + (1-\omega(1-\eta_{t+1}))c_{n,t+1}^{-\mu}\right)\right]$$

$$c_{c,t}^{-\mu} \ \geq \ \beta E_t\left[\frac{R_t}{\Pi_{t+1}} c_{c,t+1}^{-\mu}\right]$$

**Solution:** Employed Euler holds with equality and hence determines the interest rate. Unemployed and capitalist Eulers hold with inequality (i.e. borrowing constraint binds) and can be ignored.

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$$w_t^{-\mu} = \beta E_t\left[\frac{R_t}{\Pi_{t+1}}\left(\omega(1-\eta_{t+1})\vartheta^{-\mu} + (1-\omega(1-\eta_{t+1}))w_{t+1}^{-\mu}\right)\right]$$

$$c_{c,t}^{-\mu} \geq \beta E_t\left[\frac{R_t}{\Pi_{t+1}} c_{c,t+1}^{-\mu}\right]$$

**Solution:** Employed Euler holds with equality and hence determines the interest rate. Unemployed and capitalist Eulers hold with inequality (i.e. borrowing constraint binds) and can be ignored.

# Employed Euler with Endogenous Risk

Employed Euler holds with equality. Substitute in $c_{n,t} = w_t$ and $c_{u,t} = \vartheta$:

$$w_t^{-\mu} = \beta E_t \frac{R_t}{\Pi_{t+1}} \left[\omega(1-\eta_{t+1})\vartheta^{-\mu} + (1-\omega(1-\eta_{t+1}))w_{t+1}^{-\mu}\right]$$

Rewrite:

$$w_t^{-\mu} = \beta E_t \frac{R_t}{\Pi_{t+1}} w_{t+1}^{-\mu} \underbrace{\left[1+\omega(1-\eta_{t+1})\left((\vartheta/w_{t+1})^{-\mu}-1\right)\right]}_{\Theta_{t+1}}$$

This looks a lot like a normal representative agent Euler equation / New Keynesian IS Curve, except including:

$$\Theta_{t+1} = \text{endogenous earnings-risk wedge}$$

$\implies$ Precautionary motive distorts the equilibrium relative to complete markets allocation.

**Key questions:**

1. What is $\Theta_{t+1}$, and how is it determined by search frictions?
2. What is the cyclicality of $\Theta_{t+1}$?

# Understanding the endogenous earnings-risk wedge

$$w_t^{-\mu} = \beta E_t \frac{R_t}{\Pi_{t+1}} w_{t+1}^{-\mu} \underbrace{\left[1 + \omega(1 - \eta_{t+1})\left((\vartheta / w_{t+1})^{-\mu} - 1\right)\right]}_{\Theta_{t+1}}$$

[**Note:** Since agents take $w_t$ etc as given, easiest to think of this equation as determining the equilibrium interest rate $R_t$ required to clear the bond market]

**1. What is $\Theta_{t+1}$?**

- $\Theta_{t+1}$ captures the extra precautionary savings demand of employed workers due to the (uninsured) possibility of becoming unemployed next period
- If $\Theta_{t+1} = 1$ at all times, then model $\simeq$ representative agent model
- Since $w_{t+1} > \vartheta$, $\Theta_{t+1} > 1$, and moreso the higher is risk aversion ($\mu$)

**2. What is the cyclicality of $\Theta_{t+1}$?**

- Procyclical force: wage is higher in booms, so have more to lose $\implies \uparrow \Theta_{t+1}$
- Countercyclical force: less likely to lose your job in booms, so less need to save $\implies \downarrow \Theta_{t+1}$
- Overall the countercyclical force dominates both empirically and theoretically, because $\eta$ is more cyclical than $w$. Important idea: earnings risk is countercyclical!

# Full model statement

▸ firm problem and nkpc

I assume risk-neutral capitalists ($\Lambda_{t,t+1} = \beta$), rigid real wage $w_t = \bar{w}$, simplified Taylor rule for parsimony.

**(1) Worker Euler with endogenous earnings risk:** $\bar{w}^{-\mu} = \beta E_t \frac{R_t}{\Pi_{t+1}} \bar{w}^{-\mu} \left[1 + \omega(1 - \eta_{t+1}) \left((\vartheta / \bar{w})^{-\mu} - 1\right)\right]$

**(2) Marginal cost (with $v \geq 0$ multiplier, $\lambda_{v,t} \geq 0$):** $mc_t = \frac{1}{e^{A_t}} \left(\bar{w} + \frac{\kappa}{q_t} - \lambda_{v,t} - (1-\omega)\beta E_t \left[\frac{\kappa}{q_{t+1}} - \lambda_{v,t+1}\right]\right)$

**(3) Rotemberg NKPC:** $\gamma mc_t = \varphi(\Pi_t - 1)\Pi_t - \varphi\beta E_t \left[(\Pi_{t+1} - 1)\Pi_{t+1} \frac{y_{t+1}}{y_t}\right] + \gamma - 1$

**(4) Taylor rule:** $R_t = \max\left\{\bar{R}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\delta_\pi}, 1\right\}$ where $\delta_\pi > 1$, $\bar{\Pi}$ is the inflation target, and $\bar{R}$ chosen to target s.s. $\bar{\eta}$

**(5) Production function, with $A_t$ an AR(1) process:** $y_t = e^{A_t} n_t$

**(6) Vacancy filling rate:** $q_t = \theta_t^{-\alpha}$

**(7) Job finding rate:** $\eta_t = \theta_t^{1-\alpha}$

**(8) Employment dynamics:** $n_t = (1-\omega)n_{t-1} + \eta_t\left[(1 - \xi - n_{t-1}) + \omega n_{t-1}\right]$

Including the complementary slackness condition for non-negative vacancies (which can be restated as $\lambda_{v,t}\eta_t = 0$), this is 9 equations in the 9 unknown endogenous variables $R_t, \Pi_t, mc_t, \theta_t, \eta_t, q_t, n_t, y_t, \lambda_{v,t}$.

# Steady-State Graphical Analysis

# Steady-State EE Curve (Euler Equation + Taylor Rule)

Fix a deterministic steady state (drop time subscripts, $E[\cdot]$). Ignore ZLB for the lecture.

**Worker Euler with endogenous earnings risk:**

$$1 = \beta \frac{R}{\Pi} \underbrace{\left[1 + \omega(1-\eta)\left((\vartheta/\bar{w})^{-\mu} - 1\right)\right]}_{\Theta(\eta)}$$

**Taylor rule (non-ZLB):** *[assume $\delta_\pi > 1$ so Taylor Principle holds]*

$$R = \bar{R}\left(\frac{\Pi}{\bar{\Pi}}\right)^{\delta_\pi}$$

**Combine the two to make EE curve:**

$$\Pi = \beta\,\bar{R}\left(\frac{\Pi}{\bar{\Pi}}\right)^{\delta_\pi}\Theta(\eta) \qquad \Longrightarrow \qquad \Pi = \Pi^{EE}(\eta).$$

**Countercyclical risk:** $(\vartheta/\bar{w})^{-\mu} - 1 > 0 \Rightarrow \Theta'(\eta) < 0 \implies$ **Upwards slope:** $\Pi^{EE\,\prime}(\eta) > 0$

**Intuition:** $\uparrow \eta \implies \downarrow$ risk $\implies \downarrow$ precautionary saving $\implies \uparrow$ consumption demand $\implies \uparrow \Pi$

# Steady-State PC Curve (NKPC + how $\eta$ affects marginal cost)

In steady state: $y_{t+1}/y_t = 1$ and $\Pi_{t+1} = \Pi_t = \Pi$.

**Rotemberg NKPC:**

$$\gamma mc = \varphi(\Pi - 1)\Pi - \varphi\beta(\Pi - 1)\Pi + \gamma - 1 \quad \Longrightarrow \quad \Pi = \Pi^{MC}(mc) \quad \text{with} \quad \Pi^{MC\,\prime}(mc) > 0$$

**Marginal cost determined by wage + hiring costs:** *Start with $v > 0$ case, and using $q = \eta^{\frac{-\alpha}{1-\alpha}}$:*

$$mc(\eta) = \frac{1}{e^{\bar{A}}}\left(\bar{w} + (1 - (1 - \omega)\beta)\kappa\eta^{\frac{\alpha}{1-\alpha}}\right) \quad \text{with} \quad mc'(\eta) > 0$$

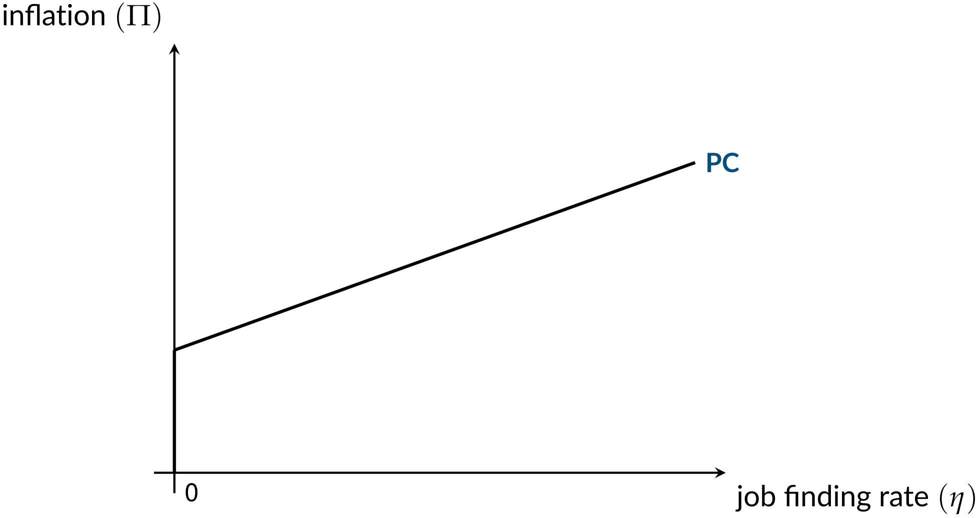
At $v = 0$ (i.e. $\eta = 0$) the $v \geq 0$ constraint binds and $mc(\eta)$ becomes vertical. Thus the PC curve is:

$$\Pi = \Pi^{MC}(mc(\eta)) = \Pi^{PC}(\eta)$$

which is upwards sloping with a vertical extension down to 0 at $\eta = 0$.

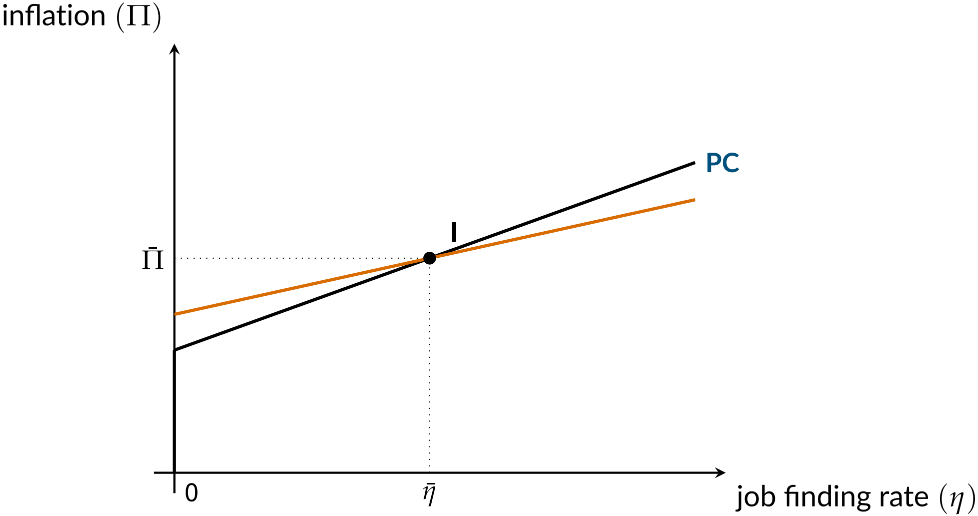
**Intuition:** $\uparrow \eta \implies \downarrow$ vacancy filling rate $\implies \uparrow$ hiring costs $\implies \uparrow$ marginal costs $\implies \uparrow \Pi$

# Steady-State Geometry: PC and EE



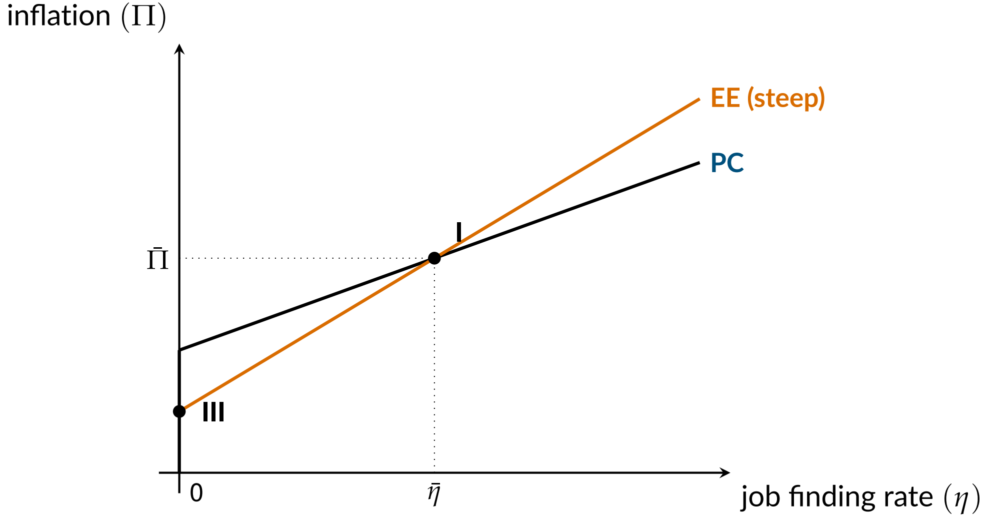

*PC curve: Upwards sloping with vertical segment when vacancies are zero at* $\eta = 0$

# Steady-State Geometry: PC and EE




*If EE is relatively flat, there is a unique steady-state at the central bank's intended equilibrium $\bar{\eta}$, $\bar{\Pi}$ (I).*

# Steady-State Geometry: PC and EE




*If EE is sufficiently steep (strong countercyclical risk; low $\delta_\pi$), a second equilibrium emerges at $\eta = 0$ (III).*

# Summary

**Key idea is a demand amplification channel via countercyclical income risk:**

Low job finding rate $\implies$ more risk $\implies$ more precautionary saving $\implies$ less demand $\implies$ less job posting $\implies$ lower job finding rate, ...

**This is summarised by the upwards sloping EE curve.** *[paper covers other cases, inc. ZLB]*

In the limit of a steep enough EE curve, this can lead to a self-fulfilling prophesy (multiple steady state equilibria)! *[Note: quite extreme, $\eta = 0 \implies$ 100% unemployment, but can relax this]*

The steeper the EE curve, the stronger the amplification. EE curve is steeper if:

- Large consumption fall on unemployment (small $\vartheta / \bar{w}$)
- More risk averse workers (high CRRA coefficient $\mu$)
- Central bank acts less strongly to stabilise demand (small $\delta_\pi$)

We focused on steady states, but this amplification idea also holds in response to temporary shocks, and in richer models with full non-degenerate wealth distributions.

# Summary

# HANK-SAM with a full asset distribution

Focused today on analytical insights from Ravn and Sterk (2021). But plenty of quantitative HANK-SAM models exist (and are becoming more common).

**Some key contributions are:**

1. Gornemann et al. (2022): possibly the first full HANK-SAM (first draft 2012, but still in R&R)
2. Challe et al. (2017): early full HANK-SAM, including full estimation
3. Ravn and Sterk (2017): similar to their analytical paper, allowing asset distribution
4. den Haan et al. (2018): households solve portfolio problem of bonds vs. stocks
5. Challe and Ragot (2016): early paper using a different limited-heterogeneity trick
6. More papers using zero-liquidity: Challe (2020), McKay and Reis (2021), Broer et al. (2025)

**An important idea: Is $\uparrow$ precautionary saving expansionary or contractionary?**

- Precautionary saving $\implies$ buy more stocks, capital, ... $\implies \uparrow I \implies \uparrow GDP$
- Precautionary saving $\implies$ buy fewer consumption goods $\implies \downarrow$ demand $\implies \downarrow GDP$
- Key question: is $\uparrow$ saving into investment, or into money/bonds? See 2 and 4 above.

# Summary of QM2 labour lectures

**Lecture 1:** *(HA+SAM)*

- Macro labour models naturally force us to think about heterogeneity
- McCall: wage heterogeneity $\Rightarrow$ theory of reservation wages
- KMS: add incomplete markets to DMP to study insurance vs job creation

**Lecture 2:** *(HANK+SAM)*

- Labour market tightness $\Rightarrow$ endogenous income risk
- Risk feeds back into aggregate demand

**Big takeaway:**

Search frictions endogenise (countercyclical) income risk in HANK models.

# Reading List

# Core Textbooks and required reading

**Core textbooks:** *[not required reading, but useful resources]*

- Pissarides (2000): *Equilibrium Unemployment Theory (2nd ed).* Classic DMP reference.
- Ljungqvist and Sargent (2018): *Recursive Macroeconomic Theory (4th ed).*
- Cahuc et al. (2014): *Labor Economics (2nd ed).* Comprehensive textbook on labor economics.
- Quant Econ coding course website, including McCall model in Julia and Python.

**The following papers are considered <u>required reading</u> for lectures 1 and 2:**

- Krusell et al. (2010) *Labour-Market Matching with Precautionary Savings and Aggregate Fluctuations*
- Ravn and Sterk (2021) *Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach*
- *[no need to memorise or perfectly understand everything, but you must at least read these]*

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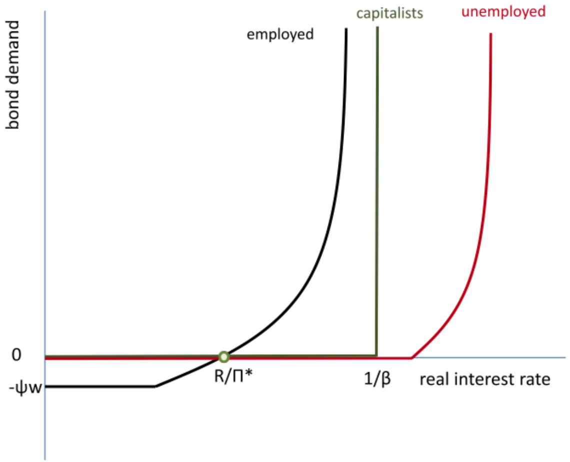
# Appendix: Ravn Sterk model

# Zero liquidity equilibrium proof sketch ‣ return




Equilibrium must take the form:

- Whoever wants to save the most, their Euler equation holds with equality when evaluated at consumption=income, so that they optimally choose to save exactly zero.
- $\implies$ this Euler equation pins down the equilibrium interest rate.
- The remaining agents, who wanted to save less, all choose to hit their borrowing constraints
- We can then ignore their Euler equations, which just determine their Lagrange multipliers

In this model:

- Unemployed want to borrow, not save
- Capitalists have no income risk so no precautionary saving motive
- Employed want to save the most, as they have a precautionary savings motive to save against future unemployment

# Firm Problem

Intermediate firm $j$ chooses $\{P_{j,t}, v_{j,t}, n_{j,t}\}_{t\geq 0}$ to maximise:

$$E_t \sum_{s=t}^{\infty} \Lambda_{t,s} \left[ \frac{P_{j,s}}{P_s} y_{j,s} - w_s n_{j,s} - \kappa v_{j,s} - \frac{\varphi}{2} \left( \frac{P_{j,s} - P_{j,s-1}}{P_{j,s-1}} \right)^2 y_s \right]$$

subject to:

$$y_{j,t} = \exp(A_t) n_{j,t}$$

$$n_{j,t} = (1-\omega) n_{j,t-1} + q_t v_{j,t}$$

$$v_{j,t} \geq 0$$

$$y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\gamma} y_t.$$

$\Lambda_{t,s}$ is the owners' real SDF.

# Vacancy FOC and Shadow Value of a Match

Let $\mathcal{J}_{j,t}$ be multiplier on employment law. Let $q_t \lambda_{v,j,t}$ be multiplier on $v_{j,t} \geq 0$.

FOC w.r.t. $v_{j,t}$:

$$-\kappa + \mathcal{J}_{j,t} q_t + q_t \lambda_{v,j,t} = 0.$$

Hence:

$$\mathcal{J}_{j,t} = \frac{\kappa}{q_t} - \lambda_{v,j,t}.$$

KKT conditions:

$$\lambda_{v,j,t} \geq 0, \quad v_{j,t} \geq 0, \quad \lambda_{v,j,t} v_{j,t} = 0.$$

Interior: $\lambda_{v,j,t} = 0$.

Corner (trap): $v_{j,t} = 0$, $\lambda_{v,j,t} > 0$.

# Marginal Value of Employment

One additional worker at $t$ yields:

$$\text{Current cost: } w_t + \mathcal{J}_{j,t}.$$

Match survives with probability $1 - \omega$.
Continuation value:

$$(1 - \omega) E_t \left[ \Lambda_{t,t+1} \mathcal{J}_{j,t+1} \right].$$

Hence marginal resource cost of one more worker:

$$w_t + \mathcal{J}_{j,t} - (1 - \omega) E_t \left[ \Lambda_{t,t+1} \mathcal{J}_{j,t+1} \right].$$

# Real Marginal Cost

Since $y_{j,t} = \exp(A_t) n_{j,t}$,

$$\frac{\partial n_{j,t}}{\partial y_{j,t}} = \frac{1}{\exp(A_t)}.$$

Thus real marginal cost:

$$mc_t = \frac{1}{\exp(A_t)} \left( w_t + \mathcal{J}_t - (1-\omega) E_t \left[ \Lambda_{t,t+1} \mathcal{J}_{t+1} \right] \right).$$

Substitute $\mathcal{J}_t = \frac{\kappa}{q_t} - \lambda_{v,t}$.:

$$mc_t = \frac{1}{\exp(A_t)} \left( w_t + \frac{\kappa}{q_t} - \lambda_{v,t} - (1-\omega) E_t \left[ \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} - \lambda_{v,t+1} \right] \right).$$

# Price Setting with Rotemberg Costs

Real profits in symmetric equilibrium:

$$\left(\frac{P_t}{P_t}\right)^{1-\gamma} y_t - w_t n_t - \kappa v_t - \frac{\varphi}{2}(\Pi_t - 1)^2 y_t.$$

Firm chooses $P_t$ taking $mc_t$ as given.

FOC w.r.t. $P_t$ yields:

$$\varphi(\Pi_t - 1)\Pi_t - \varphi E_t \left[\Lambda_{t,t+1}(\Pi_{t+1} - 1)\Pi_{t+1}\frac{y_{t+1}}{y_t}\right] = 1 - \gamma + \gamma mc_t.$$

# Nonlinear NKPC

Rearranging:

$$\gamma mc_t = \varphi(\Pi_t - 1)\Pi_t - \varphi E_t \left[ \Lambda_{t,t+1}(\Pi_{t+1} - 1)\Pi_{t+1} \frac{y_{t+1}}{y_t} \right] + \gamma - 1.$$

Inflation determined by:

- Current marginal cost.
- Expected future inflation.
- Discount factor $\Lambda_{t,t+1}$.

# Risk-Neutral Capitalists (Simplification) return

If owners are risk neutral:

$$\Lambda_{t,t+1} = \beta.$$

Marginal cost simplifies to:

$$mc_t = \frac{1}{\exp(A_t)} \left( w_t + \frac{\kappa}{q_t} - \lambda_{v,t} - (1-\omega)\beta E_t \left[ \frac{\kappa}{q_{t+1}} - \lambda_{v,t+1} \right] \right).$$

NKPC becomes:

$$\gamma mc_t = \varphi(\Pi_t - 1)\Pi_t - \varphi\beta E_t \left[ (\Pi_{t+1} - 1)\Pi_{t+1} \frac{y_{t+1}}{y_t} \right] + \gamma - 1.$$

These are the equations used in the main text.